Circular waveguides

Introduction

Waveguides can be simply described as metal pipes. Depending on their cross section there are rectangular waveguides (described in separate tutorial) and circular waveguides, which cross section is simply a circle.

This tutorial is dedicated to basic properties of circular waveguides. For properties visualisation the electromagnetic simulations in *QuickWave* software are used.

All examples used here were prepared in free CAD *QW-Modeller* for *QuickWave* and the models preparation procedure is described in separate documents. All examples considered herein are included in the QW-Modeller and QuickWave STUDENT Release installation as both, *QW-Modeller* and *QW-Editor* projects.

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Cutoff frequency

Similarly as in the case of rectangular waveguides, propagation in circular waveguides is determined by a cutoff frequency. The cutoff frequency is unique for a particular waveguide mode that is supposed to be propagating in a waveguide of a given diameter and determines the lower frequency of the waveguide's operating frequency range.

The cutoff frequency for circular waveguide is calculated using the following formula:

$$f_{c,m,n} = \frac{v}{2\pi} \beta_{c,m,n}$$

where:

v stands for a wave velocity in a medium filling the waveguide, $\beta_{c,\ m,n}$ is a cutoff phase constant which is calculated according to the formulae given in Table 1.

Table 1 Cutoff phase constant formulae for circular waveguide modes.

TE (H) mode (*Transverse Electric*)	TM (E) mode (*Transverse Magnetic*),
$\beta_{c,m,n} = \frac{\chi'_{m,n}}{a}$	$\beta_{c,m,n} = \frac{\chi_{m,n}}{a}$

where:

$\chi_{m,n}$ – n-th root of m-th Bessel function,

$\chi'_{m,n}$ – n-th root of the m-th Bessel function derivative ,

a– radius of the circular waveguide.

Several Bessel functions and Bessel functions derivatives are shown in Fig. 1 and Fig. 2.

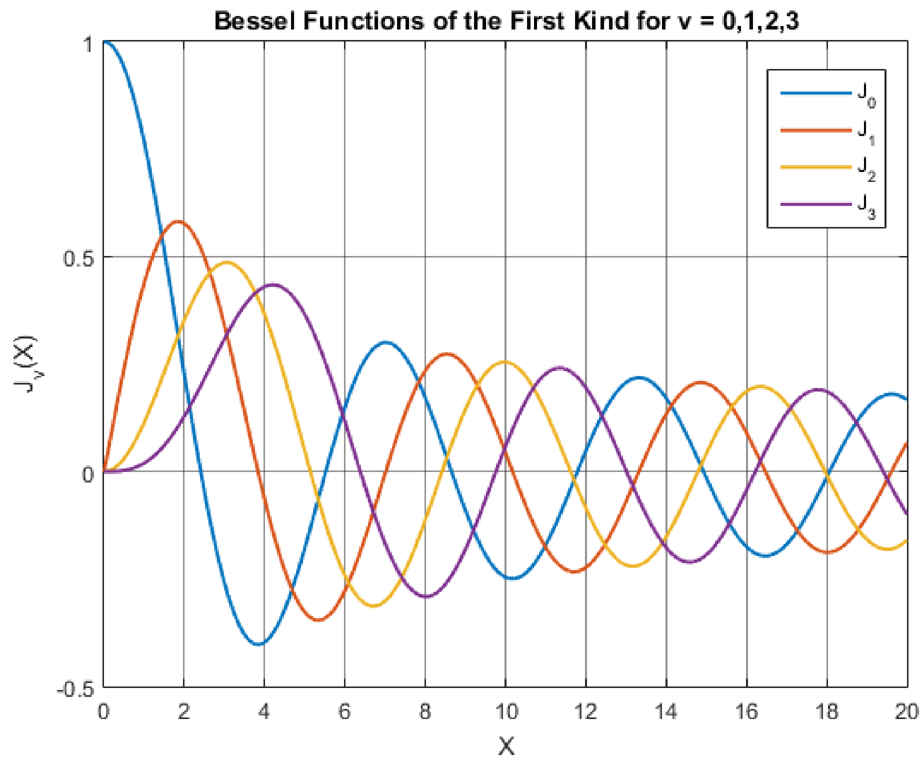


Fig. 1 Bessel functions of the first kind

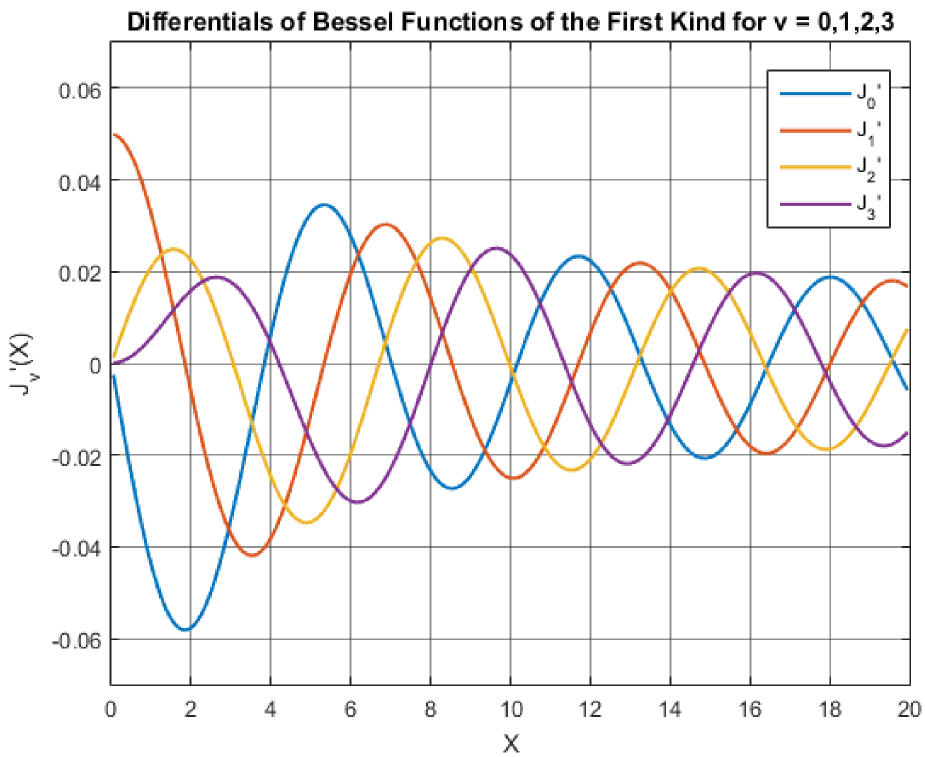


Fig. 2 Derivatives of Bessel functions of the first kind.

For the engineers' convenience the values of Bessel functions and Bessel functions derivatives are commonly given in tables (see Table 2).

Table 2 Values of Bessel functions and Bessel functions derivatives.

Function number m	**Root number** n	**Roots of the Bessel function** J_m $\chi_{m,n}$	**Roots of the Bessel function derivatives** J'_m $\chi'_{m,n}$
0	1	2,405	3,832
0	2	5,520	7,016
0	3	8,654	10,173
1	1	3,832	1,841
1	2	7,016	5,331
2	1	5,136	3,054
2	2	8,417	6,706
3	1	6,380	4,201

As an example, the cutoff frequencies of the TE_{11} and TM_{01} modes in the circular waveguide with radius of *a*=10 cm, filled with air can be calculated as follows:

TE_{11} mode:

$$f_{c,TE_{m,n}} = \frac{c}{2\pi}\frac{\chi'_{m,n}}{a} \Rightarrow f_{c,TE_{11}} = \frac{c}{2\pi}\frac{\chi'_{1,1}}{a}$$

$$\chi'_{1,1} = 1.841$$

$$f_{c,TE_{11}} = \frac{c}{2\pi}\frac{1.841}{a} = 879MHz$$

TM_{01} mode:

$$f_{c,TM_{m,n}} = \frac{c}{2\pi}\frac{\chi_{m,n}}{a} \Rightarrow f_{c,TM_{01}} = \frac{c}{2\pi}\frac{\chi_{0,1}}{a}$$

$$\chi_{0,1} = 2.405$$

$$f_{c,TM_{01}} = \frac{c}{2\pi}\frac{2.405}{a} = 1148MHz$$

Propagation modes in circular waveguide

Each waveguide mode is described by unique distribution of transverse and longitudinal components of the electric and magnetic fields. Similarly to rectangular waveguides, two kinds of waveguide modes are recognised in case of circular waveguides: TE and TM. The waveguide mode in circular waveguide is described with *m* and *n* indexes, which stand for the field variation in radial and axial directions respectively.

In case of circular waveguides the fundamental mode is TE_{11}.

Mode TE_{11}

In this part, the distribution of transverse and longitudinal fields' components of TE_{11} mode is investigated. For fields' visualisation the *QuickWave* software is used.

The circular waveguide with radius of 10 cm and the length of 30 cm is considered. The model of such waveguide *cir.QWpro* can be loaded in *QW-Modeller*. The cutoff frequency of TE_{11} mode in this waveguide is 0.879 GHz. The waveguide is excited at 1 GHz and its length is around half of a guide wavelength.

Run the electromagnetic simulation with QuickWave using *Start* 

button in *Simulation* tab (Fig. 3).

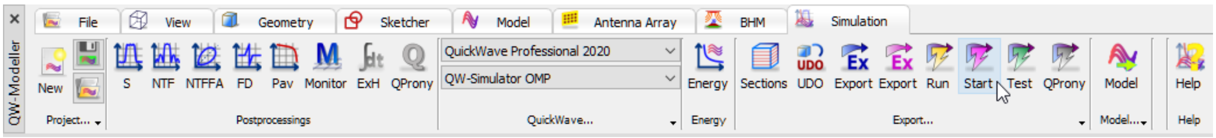
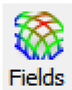


Fig. 3 Simulation tab in QW-Modeller.

Press 

button in *2D/3D Fields* tab of *QW-Simulator* to open fields visualisation window. The consecutive displays shown in Figs. 4-9 may be viewed by pressing 

button for several times (once for obtaining each of the following displays). For the visualisation convenience the display windows may be maximised.

In case of TE_{11} mode, both radial and axial components of transverse fields exists (*m*, *n* idexes are non-zero), resulting in the distribution of total electric and magnetic field in the waveguide's cross section as shown in Fig. 4 and Fig. 5 respectively. The following pictures show the displays of electric and magnetic fields along the waveguide. It is clearly seen that there is no longitudinal component (in the direction of wave propagation – Z direction) of the electric field and there is longitudinal magnetic field only in this case. The displays confirm the waveguide's length to be half of guide wavelength since one wave half can be recognised along waveguide.

The fields' distribution displayes are given for a randomly chosen time moment. When the simulation results are observed on-line the fields variations as the wave is passing the waveguide will be observed.

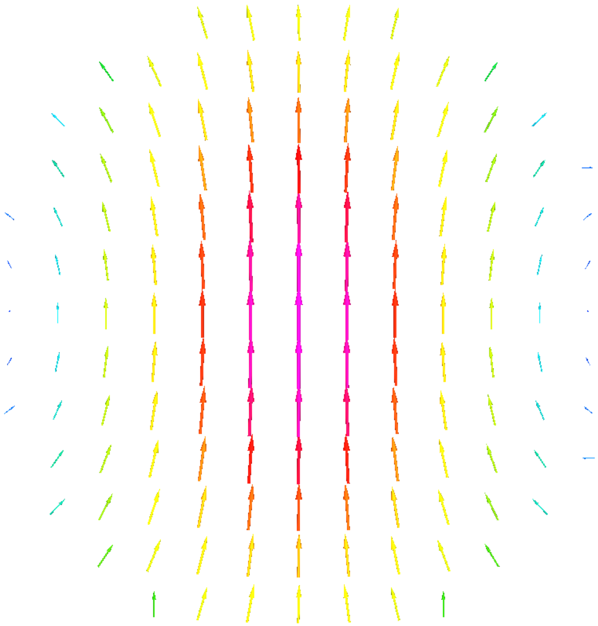


Fig. 4 A distribution of electric field for TE11 mode in a cross section of circular waveguide (YX plane).

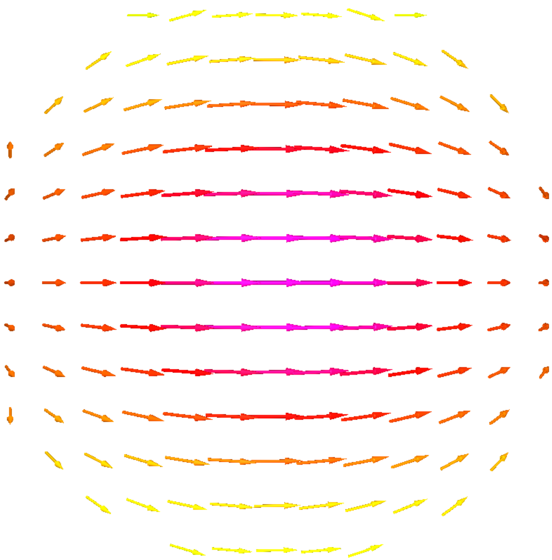


Fig. 5 A distribution of magnetic field for TE11 mode in a cross section of circular waveguide (YX plane).

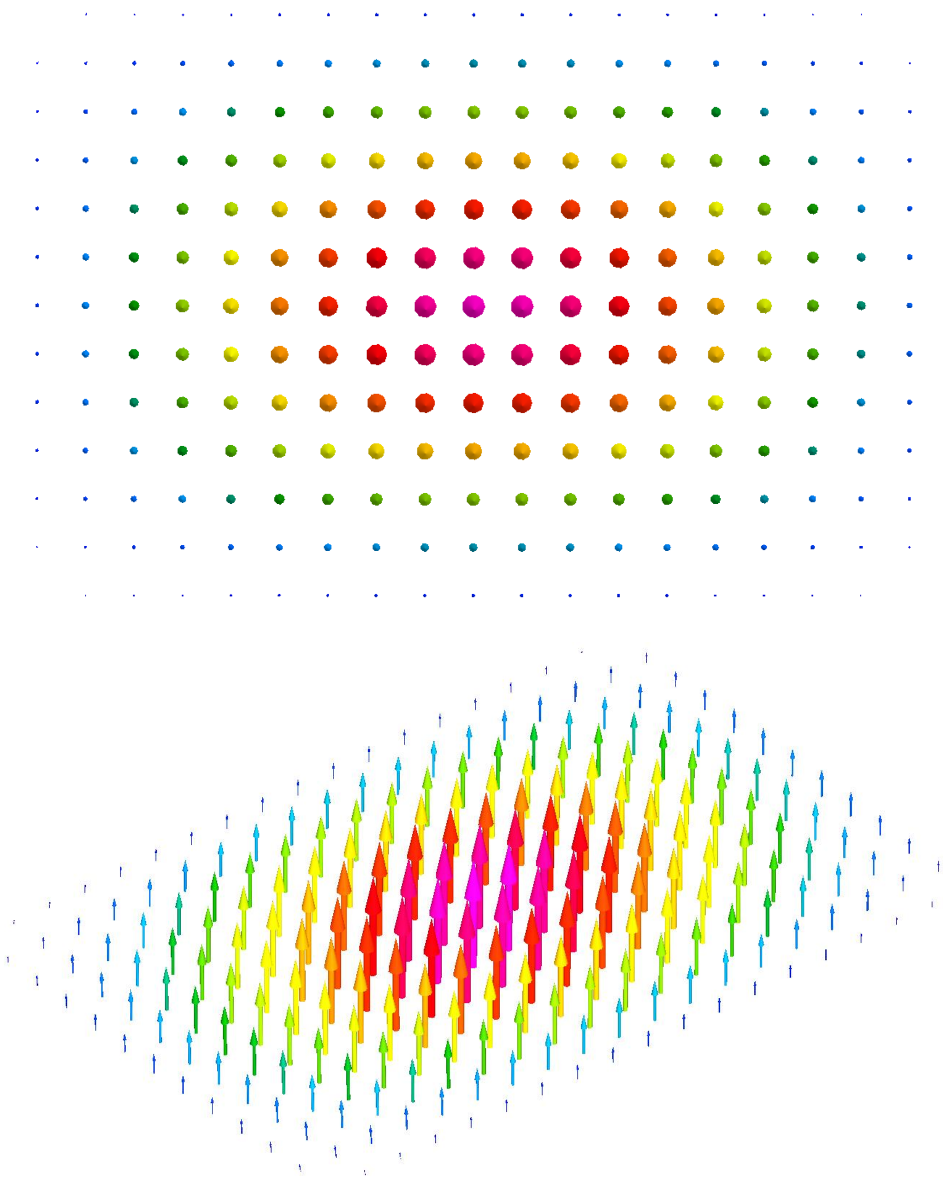


Fig. 6 A distribution of electric field for TE11 mode in circular waveguide (XZ plane – in the middle of the waveguide).

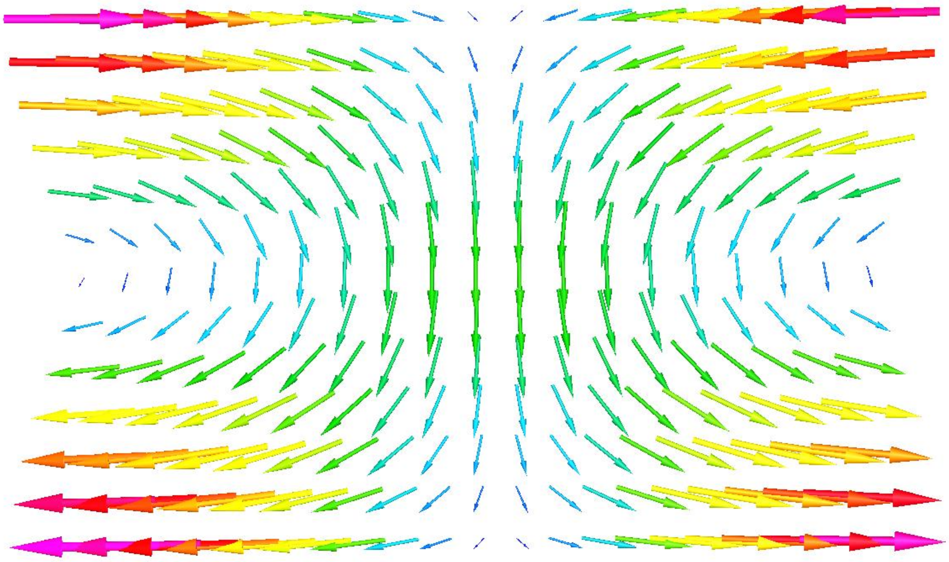


Fig. 7 A distribution of magnetic field for TE11 mode in circular waveguide (XZ plane – in the middle of the waveguide).

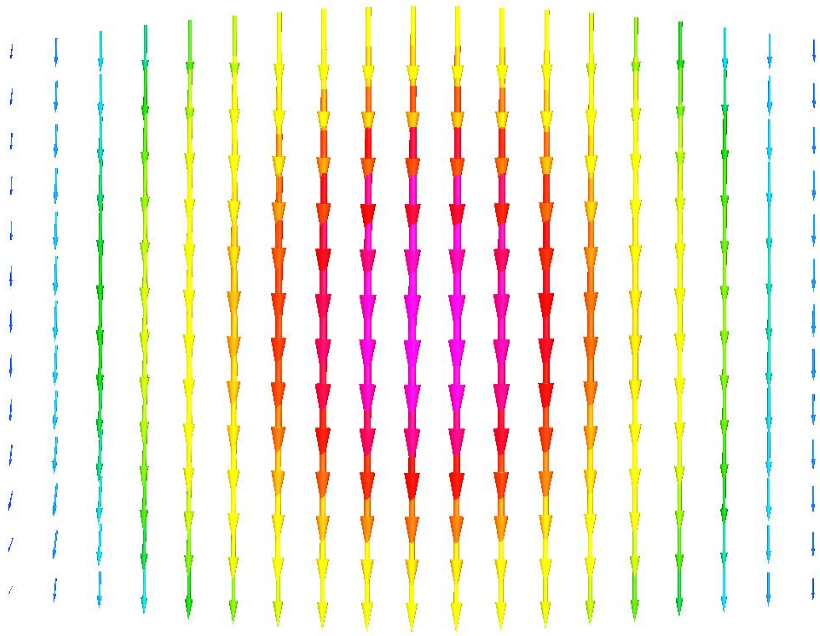
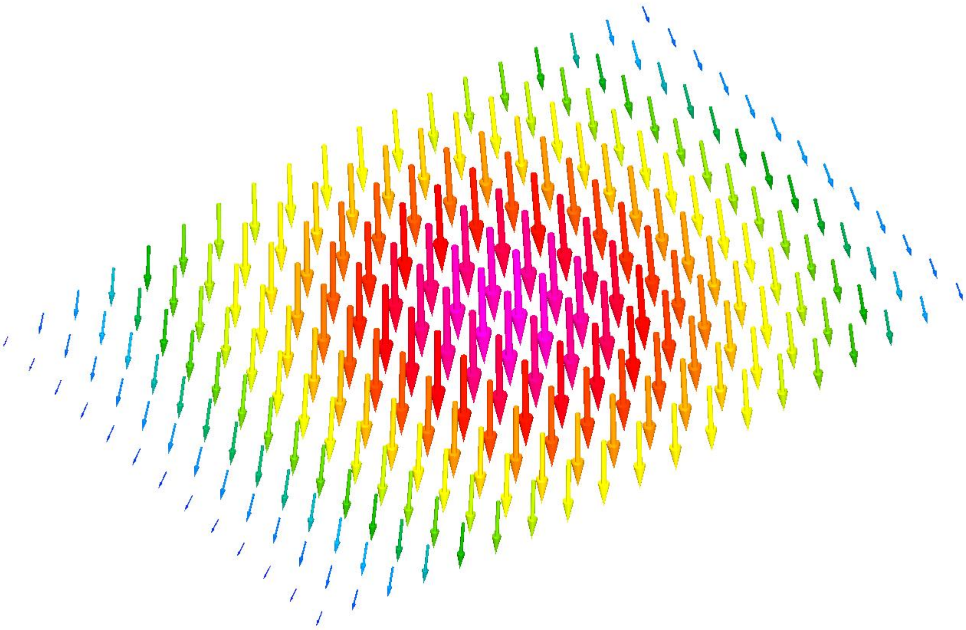
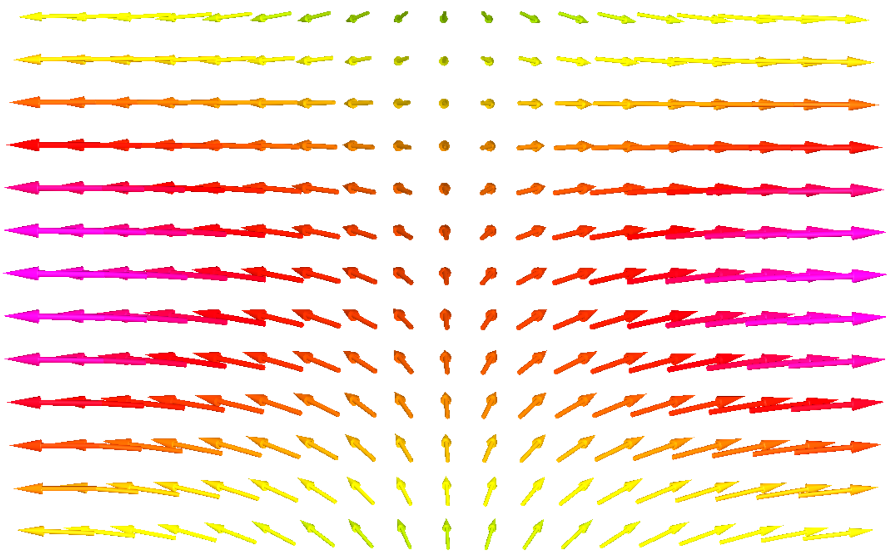


Fig. 8 A distribution of electric field for TE11 mode in circular waveguide (YZ plane – in the middle of the waveguide).



(a)



(b)

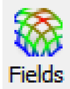
Fig. 9 A distribution of magnetic field for TE11 mode in circular waveguide (YZ plane): in the middle of the waveguide (a) and near the waveguide wall (b).

Mode TM_{01}

In this part of the tutorial the fields' distribution for the TM_{01} mode in a circular waveguide is presented. As previously, the waveguide in 30 cm long and its radius is 10 cm. The cutoff frequency for the considered mode is 1.148 GHz, thus the waveguide is excited at 2 GHz, which is above the cutoff frequency. The length of the waveguide corresponds to 1.5 of guide wavelength.

The waveguide model is contained in *cir_TM.QWpro* scenario. For fields visualisation the model can be loaded in *QW-Modeller*, from where the electromagnetic simulation in QuickWave can be run.

Press 

button in *2D/3D Fields* tab of *QW-Simulator* to open fields visualisation window. The consecutive displays shown in Figs. 10-13 may be viewed by pressing 

button for several times (once for obtaining each of the following displays). For the visualisation convenience the display windows may be maximised.

The TM mode means that there is no magnetic field component in the direction of wave propagation. The values of *m* and *n* indexes indicate that the transverse magnetic field has only an axial component (*n*=1), thus the transverse electric field has only a radial component. Fig. 10 and Fig. 11 show the transverse electric and magnetic fields' distributions respectively. Fig. 12 and Fig. 13 present the distribution of the electric and magnetic fields along the waveguide in ZX plane (cross section in the middle of the waveguide). It can be clearly seen that only the electric field has a longitudinal component (along the direction of wave propagation). It is also well visible that the waveguide's length is 1.5 of guide wavelength since three wave halves can be recognised in the fields' distributions in ZX plane. The fields' distributions in ZY plane are the same as in ZX plane since the TM_{01} mode is characterised by an axial symmetry.

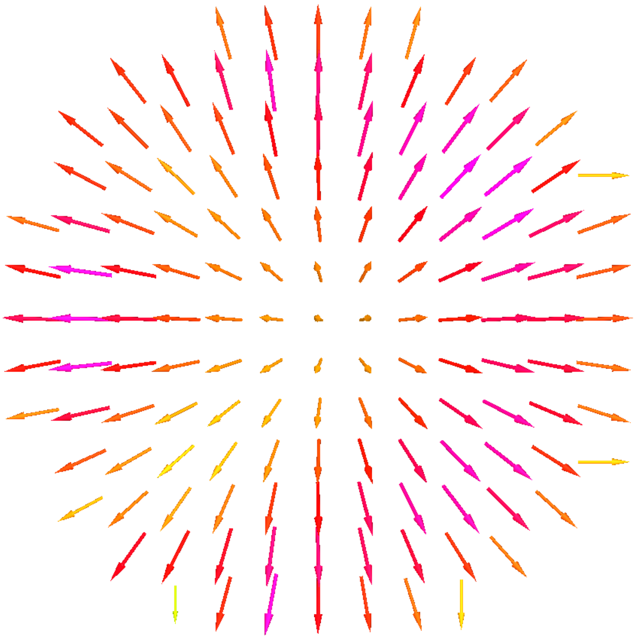


Fig. 10 A distribution of electric field for TM01 mode in a cross section of circular waveguide (YX plane).

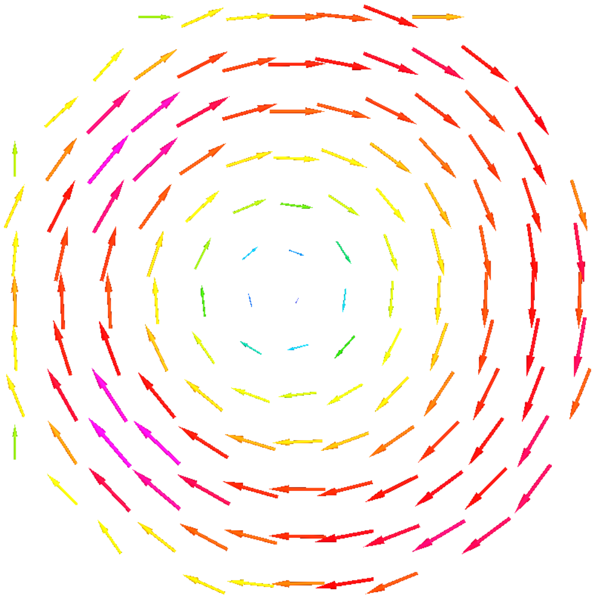


Fig. 11 A distribution of magnetic field for TM01 mode in a cross section of circular waveguide (YX plane).

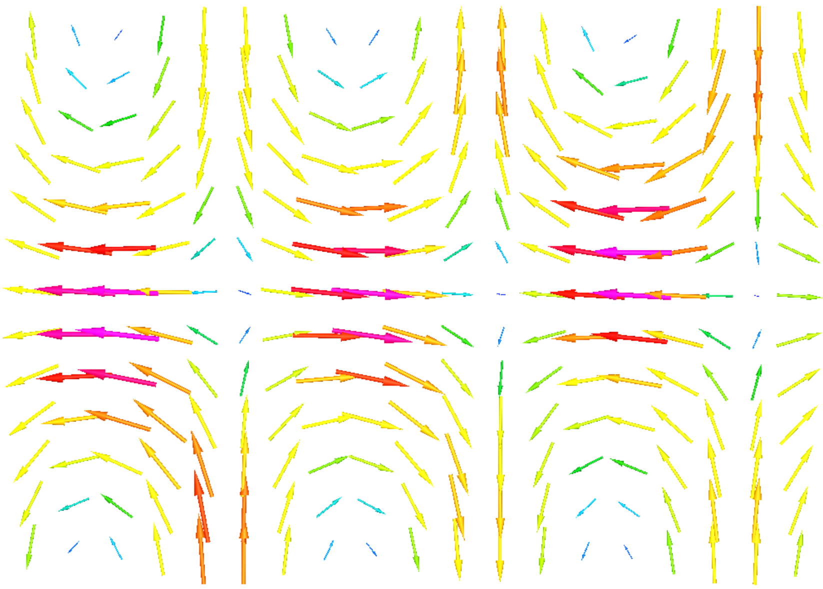
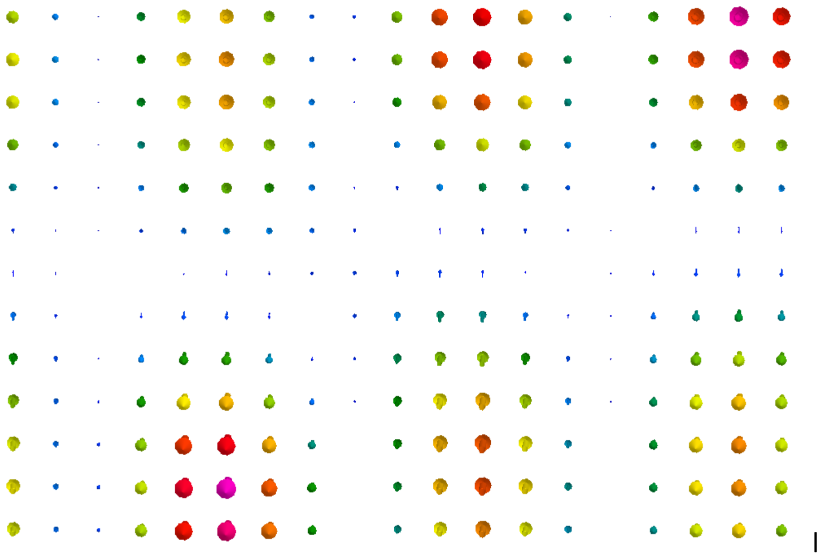


Fig. 12 A distribution of electric field for TM01 mode along the circular waveguide (ZX plane – in the middle of the waveguide).



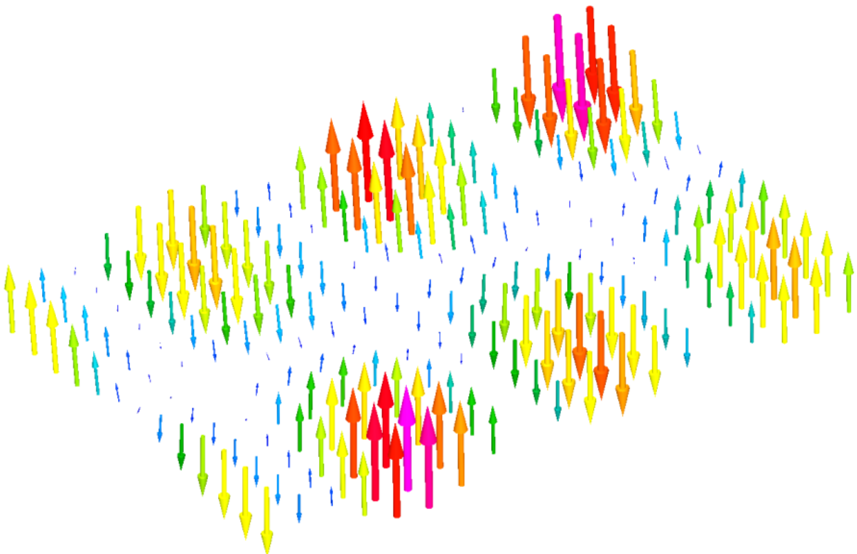


Fig. 13 A distribution of magnetic field for TM01 mode along the circular waveguide (ZX plane – in the middle of the waveguide).