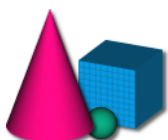# Rectangular waveguides

## Introduction

Waveguides are transmission lines commonly used in electronics, especially in higher frequency ranges like microwaves. A waveguide can be simply described as a metal pipe usually filled with air.

This tutorial is dedicated to rectangular waveguides and contains all basic information concerning their operation rules. All the interesting features are visualised using electromagnetic simulation with *QuickWave* software.

All examples used here were prepared in free CAD *QW-Modeller* for *QuickWave* and the models preparation procedure is described in separate documents. All examples considered herein are included in the QW-Modeller and QuickWave STUDENT Release installation as both, *QW-Modeller* and *QW-Editor* projects.

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## Wave propagation in rectangular waveguide

The wave is an electromagnetic wave if the Maxwell equations are obeyed. This allows defining a set of wave equations, which for an electric field is described with the following formula:

$$\nabla^2 \vec{E} - \gamma \vec{E} = 0$$

Solving this equation under the boundary conditions as in a waveguide results in the following relation:

$$\beta_\perp^2 + \beta_z^2 = \beta^2 = \omega^2 \mu \varepsilon$$

It defines the phase constant (in a lossless case it is equal to the propagation constant $\gamma$) as a sum of squares of propagation constant in the direction of propagation $\beta_z$ and a $\beta_\perp$ constant. The $\beta_\perp$ constant is strictly related to the waveguide dimensions and the waveguide mode that is propagating:

$$\beta_\perp^2 = \beta_x^2 + \beta_y^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

where $m$ and $n$ stand for waveguide mode indexes (corresponding to a number of wave halves existing along $a$ and $b$ waveguide sides respectively), $a$ and $b$ are the width and height of a waveguide.

It needs to be noted that for a given waveguide dimensions and waveguide mode ($m,n$ indexes), propagation along the waveguide will be possible only in a specific frequency range. The lower frequency of that range is called cut-off frequency and can be determined in the following way:
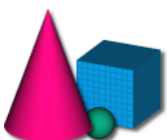
$$\beta_z^2 = \beta^2 - \beta_\perp^2$$

$$0 = \beta_c^2 - \beta_\perp^2$$

$$\beta_{c,mn}^2 = \beta_\perp^2 \Leftrightarrow \omega_{c,mn}^2 \mu \varepsilon = \beta_\perp^2 \Leftrightarrow 4\mu\varepsilon\pi^2 f_{c,mn}^2 = \beta_\perp^2 \Leftrightarrow f_{c,mn}^2 = \frac{\beta_\perp^2}{4\mu\varepsilon\pi^2} = \frac{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}{4\mu\varepsilon\pi^2}$$

$$f_{c,mn} = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

It is clearly seen that the cut-off frequency strictly depends on parameters of a medium filling the waveguide ($\mu,\varepsilon$), waveguide dimensions ($a,b$) and waveguide mode indexes ($m,n$).

## Waveguide mode

Waveguide mode stands for a unique distribution of transverse and longitudinal components of the electric and magnetic fields. There are two types of waveguide modes that can propagate in the waveguides: TE (Transverse Electric) and TM (Transverse Magnetic). In TE modes only transverse (perpendicular to the direction of propagation) electric field exists and there is no longitudinal (along propagation direction) component, and the magnetic field exists in all directions. For TM modes we deal with only transverse magnetic field and the electric field exists in all directions.

At this point it is also worth mentioning about a fundamental waveguide mode. The fundamental waveguide mode is the wave mode with the lowest cutoff frequency. In case of rectangular waveguides it is $TE_{10}$ (called also $H_{10}$ since a longitudinal component of magnetic field exists).

The waveguide modes will be investigated with *QuickWave* software. Run *QW-Modeller* and load *wr340.QWpro* project containing rectangular waveguide WR-340. The default excitation settings for this scenario can be viewed after unfolding *Ports* element group in the project tree (see Fig. 1) and editing the settings of available ports. The *SourcePort* is an excitation port delivering the energy to the waveguide. The *LoadPort* is a matched load, in which the energy is absorbed. Open the *SourcePort* settings window by double clicking on its name. The dialogue as in Fig. 2 will appear.
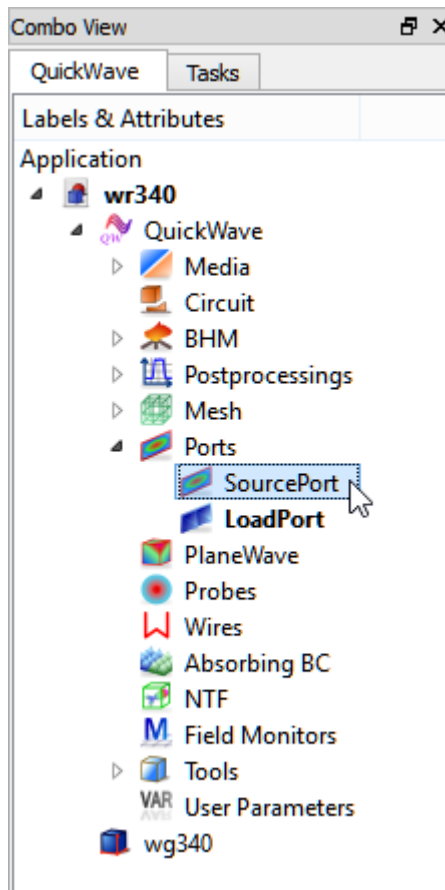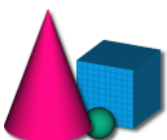


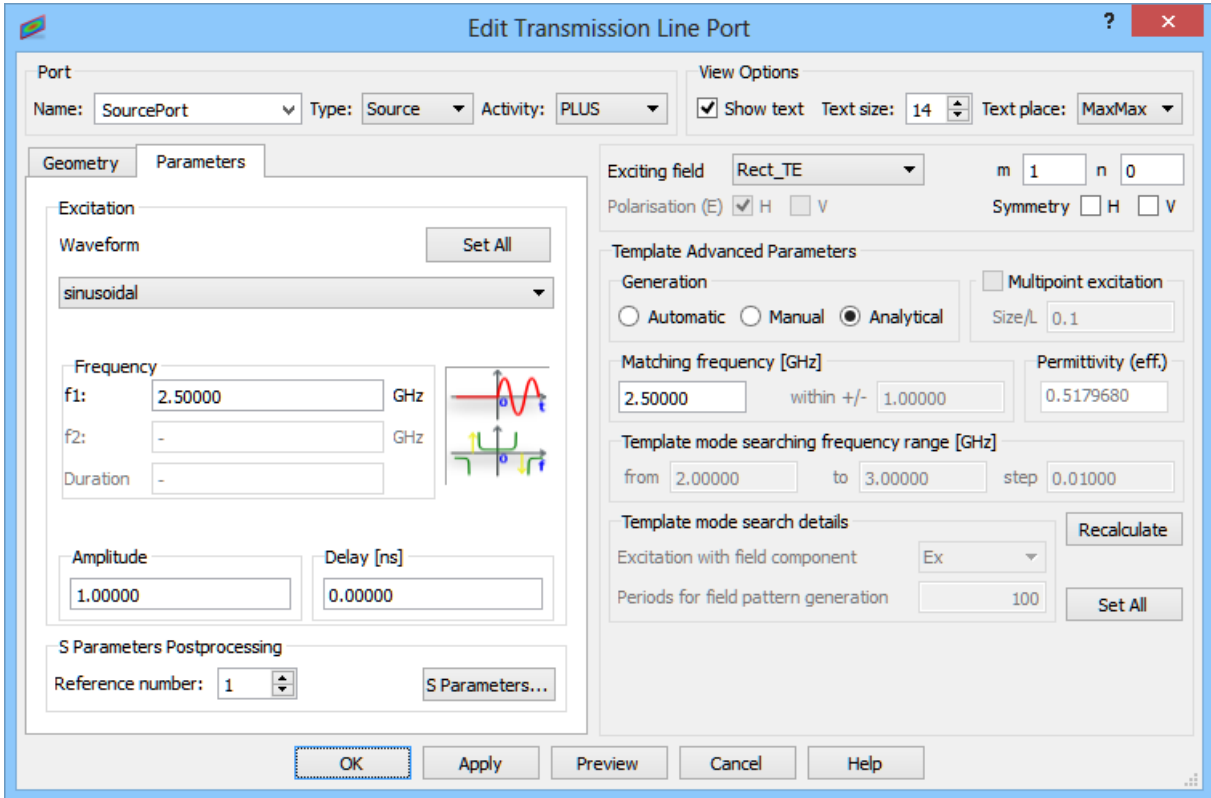*Fig. 1 Project tree of wr340.QWpro*

*Fig. 2 Port settings dialogue for SourcePort port*

In *Template Advanced Parameters* the analytical option for template mode generation is enabled. The waveguide mode type to be generated is chosen from the *Exciting field* list to be *Rect_TE*, what stands for TE mode in rectangular waveguide, and the dominant mode $TE_{10}$ is chosen by setting *m=1* and *n=0*. In the *Parameters* tab the *Excitation Waveform* is chosen to be sinusoidal and frequency is set to 2.5 GHz, which is above the cutoff frequency (1.73 GHz) for a $TE_{10}$ mode in a WR-340 waveguide. The *LoadPort* has the same excitation parameters.

For the investigation of fields' distribution, the electromagnetic simulation in *QuickWave Simulator* should be run from *Simulation* tab using *Start* button as shown in Fig. 3.
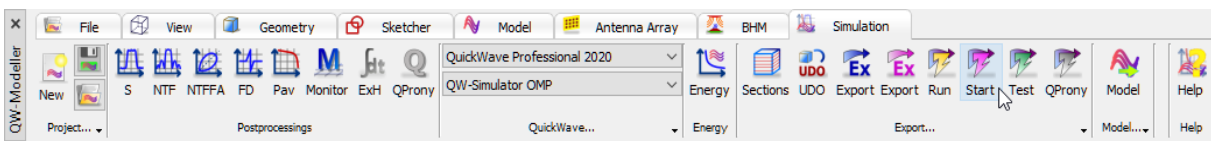


*Fig. 3 Simulation tab in QW-Modeller*

The *QW-Simulator* window with *Simulator Log* window and *Log Output* tab active will open. The description of all the information written in the log tab can be found in QuickWave manual.

Figures 4-10 show the distribution of the electric and magnetic fields for $TE_{10}$ mode in the analysed waveguide. The consecutive displays may be viewed by pressing **Fields** button in *2D/3D Fields* tab of *QW-Simulator* for several times (once for obtaining each of the following displays). For the

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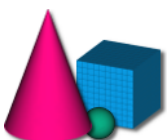
visualisation convenience the display windows may be maximised. It can be clearly seen (Fig. 4) that electric field reaches its maximum in the middle of waveguide's width and zero at the side walls. Thus one half of a sinusoid can be recognised along waveguide's width (*m=1*) and no field variation along waveguide's height (*n=0*). From the displays it is clearly seen that only magnetic field has a longitudinal component (along direction of wave propagation – X direction), which confirms that the propagating mode is a TE mode.
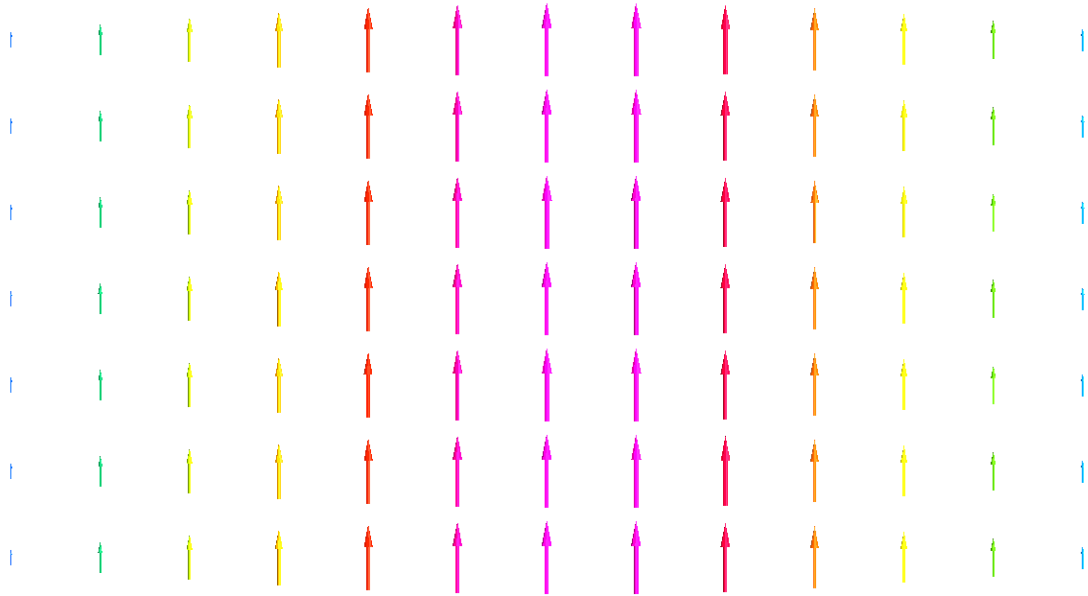


*Fig. 4 A vector display of electric field distribution for TE10 mode in a cross section of rectangular waveguide (ZY plane).*
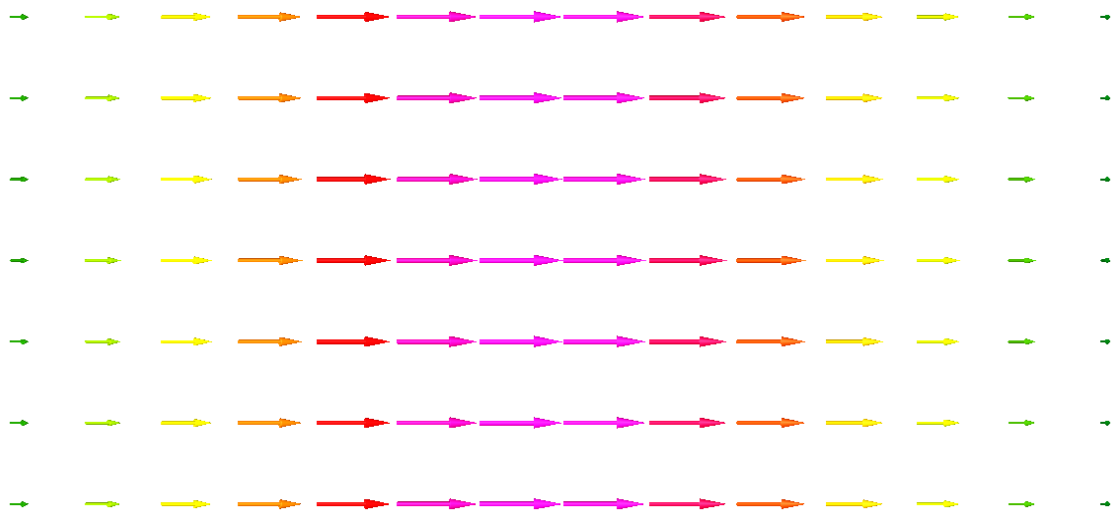


*Fig. 5 A vector display of magnetic field distribution for TE10 mode in a cross section of rectangular waveguide (ZY plane).*
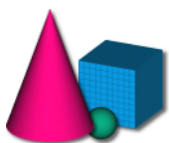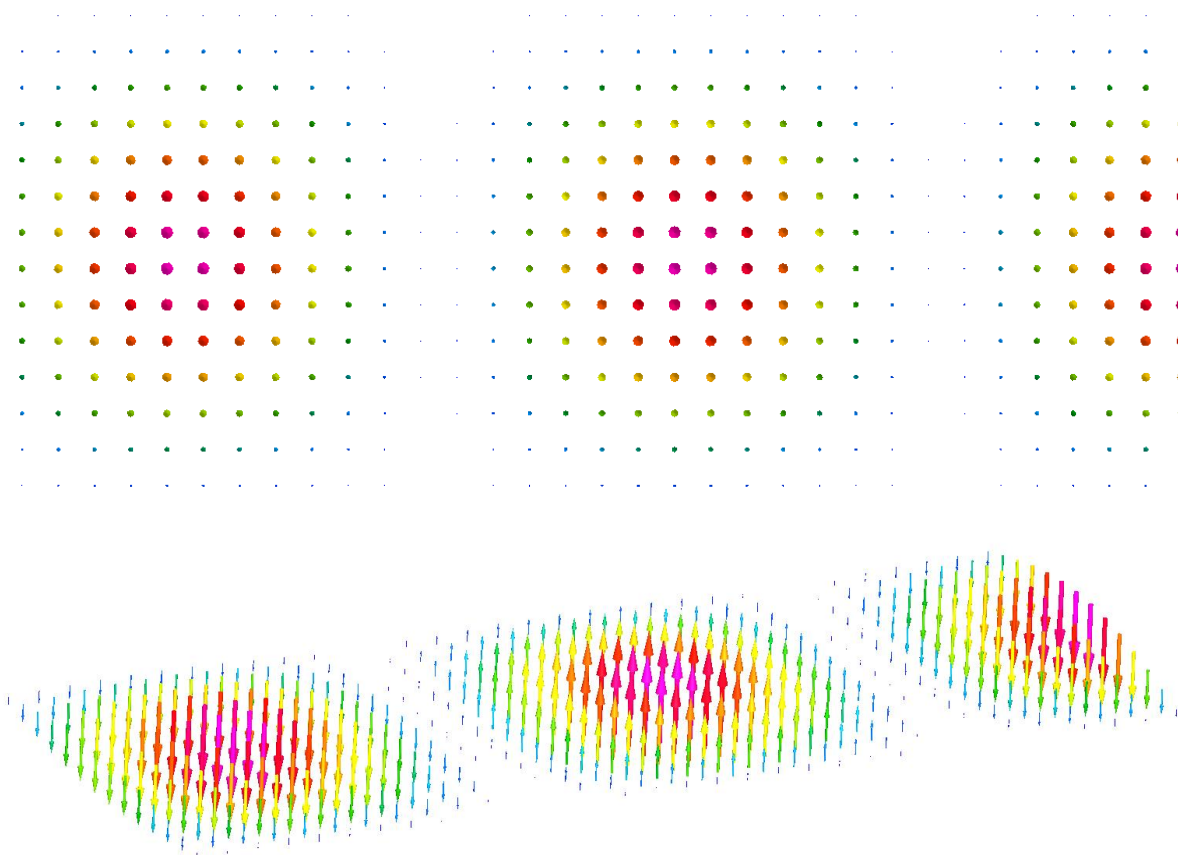
*Fig. 6 A vector display of electric field distribution for TE10 mode along rectangular waveguide (YX plane- in the middle of waveguide's height)*
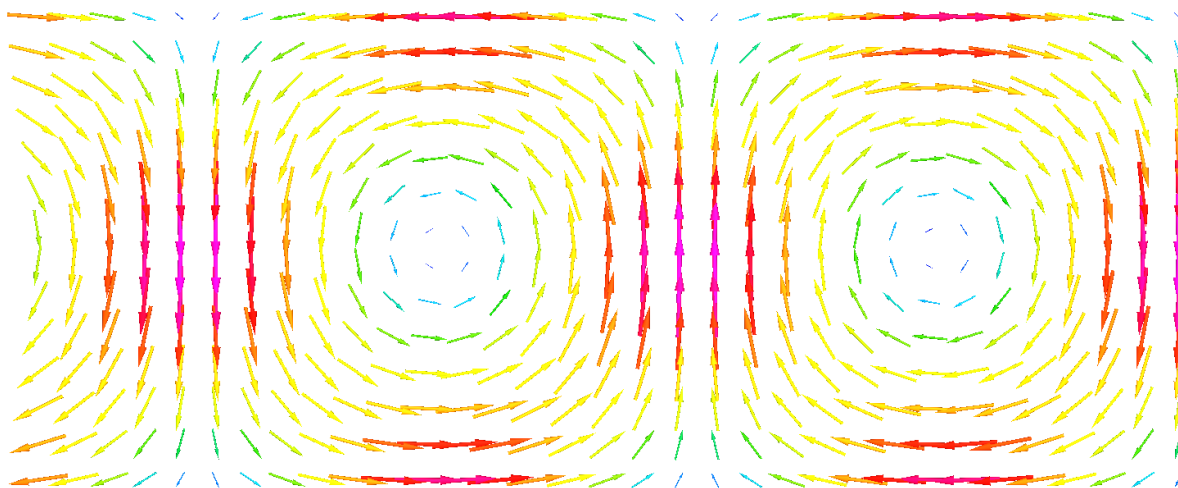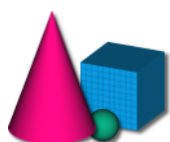


*Fig. 7 A vector display of magnetic field distribution for TE10 mode along rectangular waveguide (YX plane- in the middle of waveguide's height)*
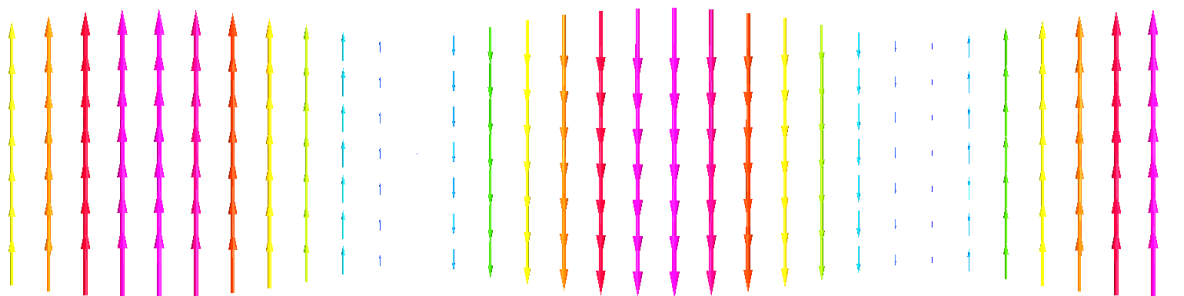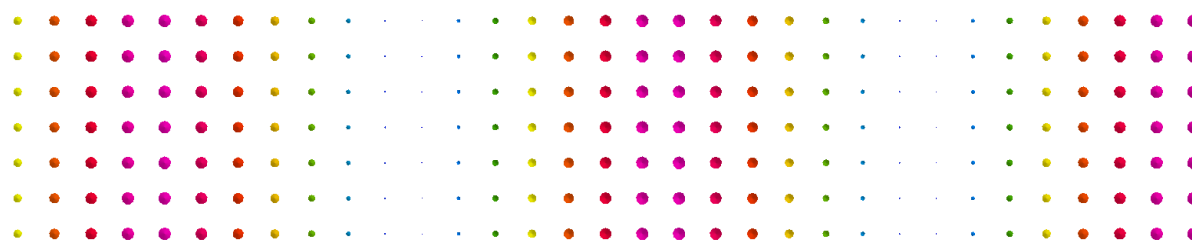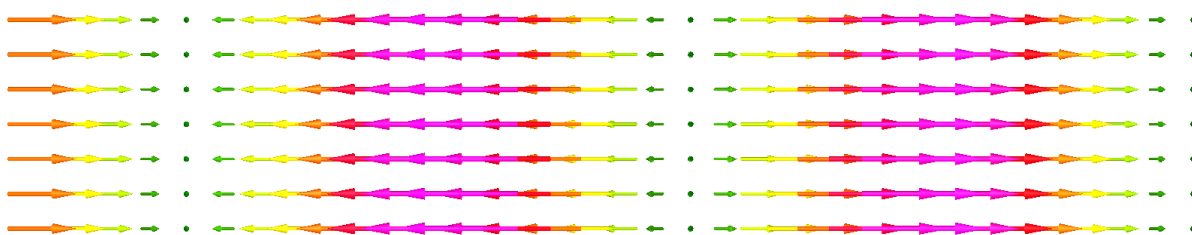
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*Fig. 8 A vector display of electric field distribution for TE10 mode along rectangular waveguide (ZX plane- in the middle of waveguide's width)*
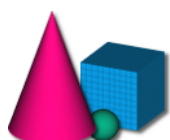
(a)

(b)

*Fig. 9 A vector display of magnetic field distribution for TE10 mode along rectangular waveguide (ZX plane): in the middle of the waveguide's width (a) and near the waveguide's wall (b).*
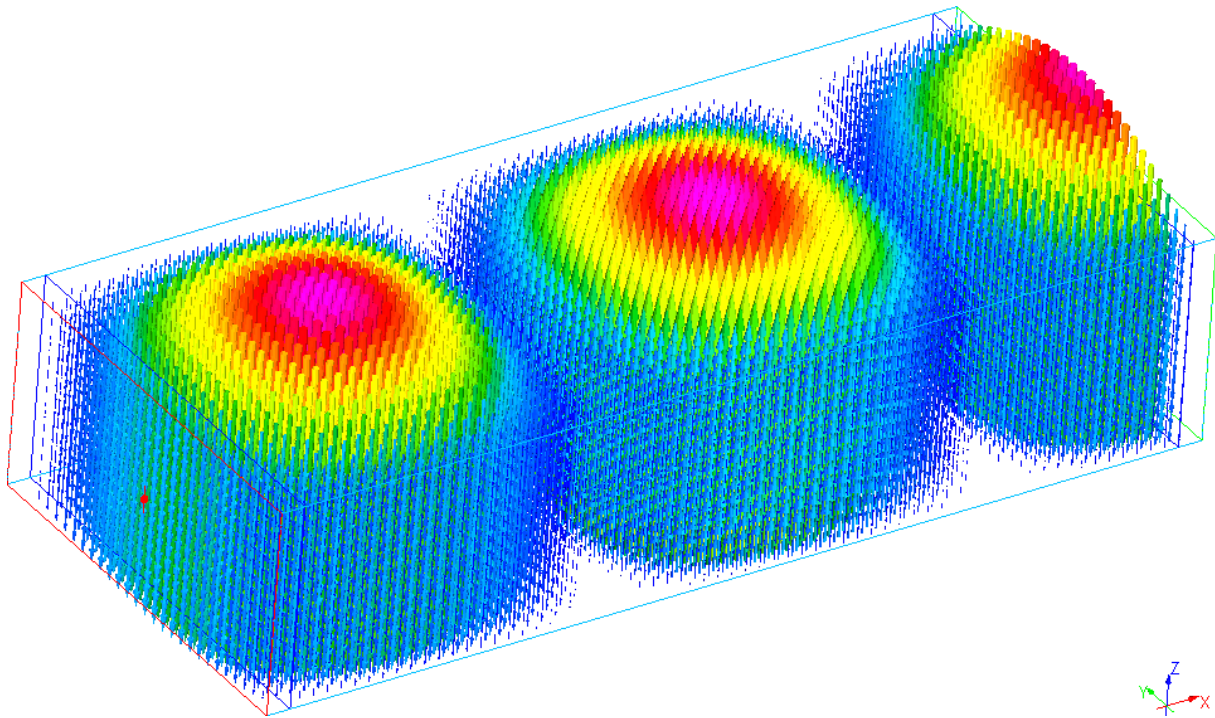
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*Fig. 10 A vector display of electric field distribution for TE10 mode along rectangular waveguide.*

## Higher order modes

In this part it is recommended to excite one of the higher order modes to feel confident in understanding and recognition of waveguide modes.

Load *wr340_TM.QWpro* model in *QW-Modeller*. In this case the TM43 mode is considered.

Note that when the waveguide mode is changed, the excitation frequency needs to be modified, so that it is above the cutoff frequency for a given waveguide. In case of the considered $TM_{43}$ mode the cutoff frequency is:

$$f_{c,TM_{43}} = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = 12.5 GHz$$

In this case the excitation frequency is set to 15 GHz. Note that the *Matching frequency* parameter available in *Template Advanced Parameters* frame should be always set to be above the cutoff frequency. Here for the convenience it is also set to 15 GHz (more information about the *Matching frequency* and the template generation procedure can be found in *QuickWave* manual). Figures 11-17 show the distribution of the electric and magnetic fields for $TM_{43}$ mode in the analysed waveguide

at 15 GHz. The consecutive displays may be viewed by pressing Fields button in *2D/3D Fields* tab of *QW-Simulator* for several times (once for obtaining each of the following displays). For the visualisation convenience the display windows may be maximised. It can be seen (Figs. 11 and 12) that there are four sinusoid halves recognised along waveguide's width (*m=4*) and three sinusoid
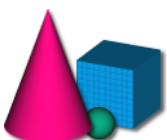
halves along waveguide's height (*n=3*). It is also well noticed that only the electric field has a non-zero component along direction of wave propagation (X direction).

It is recommended to switch between FDTD layers in each observation plane (using *L* and *K* keyboard buttons for going up and down along layers respectively) to observe how the field distribution changes along waveguide's dimensions.
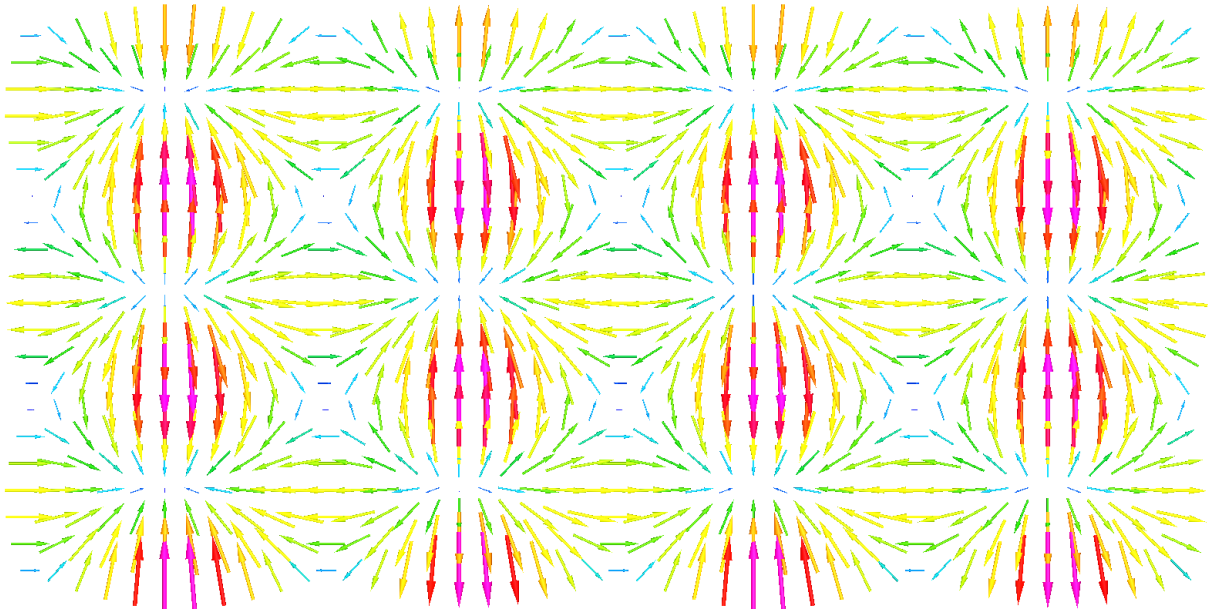


*Fig. 11 A vector display of electric field distribution for TM43 mode in a cross section of rectangular waveguide (ZY plane).*
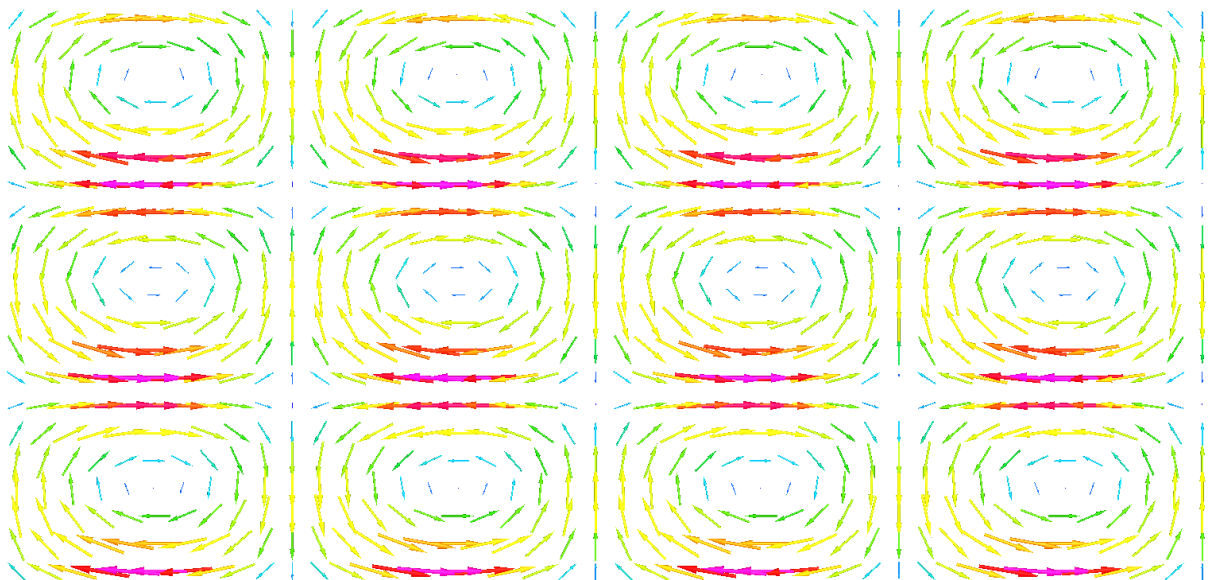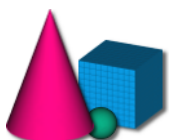


*Fig. 12 A vector display of magnetic field distribution for TM43 mode in a cross section of rectangular waveguide (ZY plane).*
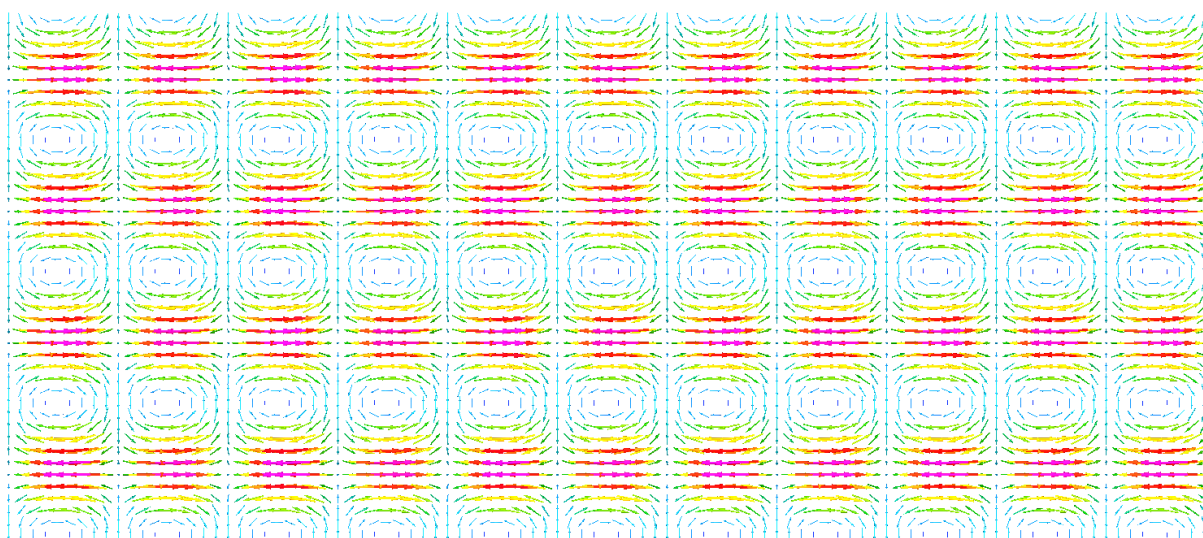
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*Fig. 13 A vector display of electric field distribution for TM43 mode along rectangular waveguide (YX plane- in the middle of waveguide's height).*
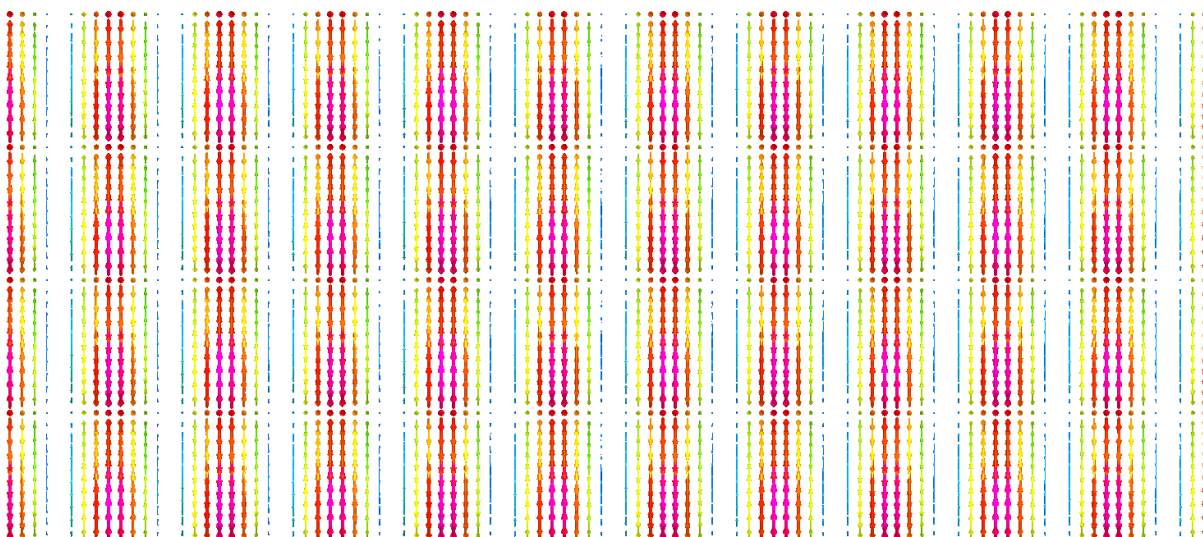


*Fig. 14 A vector display of magnetic field distribution for TM43 mode along rectangular waveguide (YX plane- in the middle of waveguide's height).*
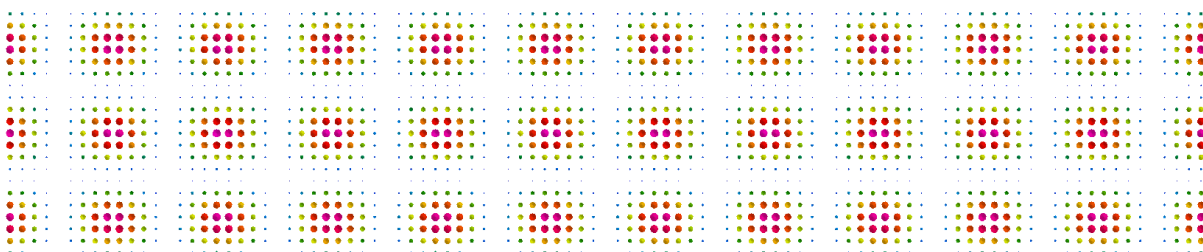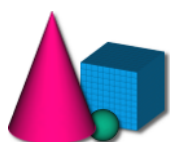


*Fig. 15 A vector display of electric field distribution for TM43 mode along rectangular waveguide (ZX plane- in the middle of waveguide's width).*
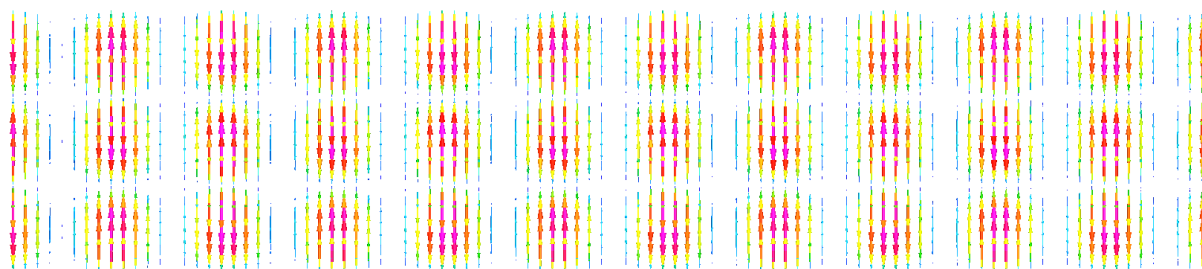
*Fig. 16 A vector display of magnetic field distribution for TM43 mode along rectangular waveguide (ZX plane- in the middle of waveguide's width).*
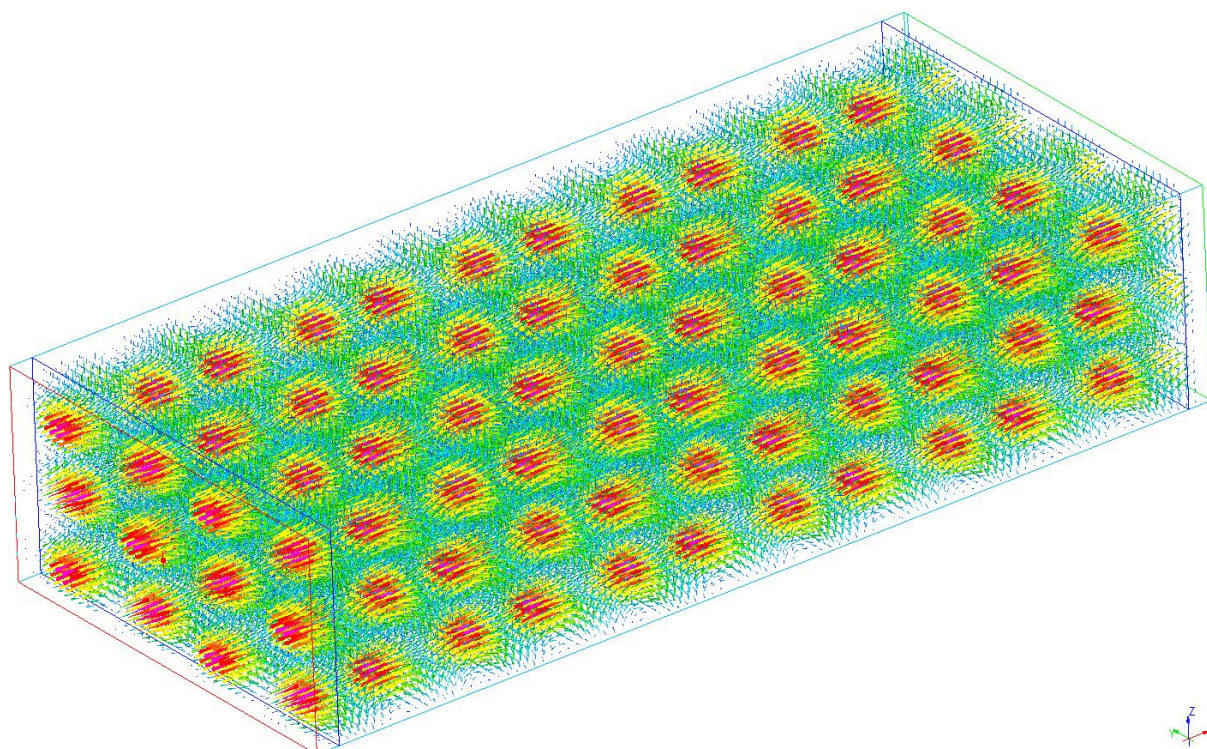


*Fig. 17 A vector display of electric field distribution for TM43 mode along rectangular waveguide.*

## Wavelength in a waveguide

It is known from Plane Wave tutorial that the wavelength of a plane wave is strictly related to the wave frequency. The wavelength in a waveguide is considered as a wavelength in a direction of wave propagation and its dependence on wave frequency is defined as follows:

$$\lambda_z = \frac{\lambda_0}{\sqrt{1 - \left(\dfrac{\lambda_0}{\lambda_c}\right)^2}}$$

where $\lambda_0$ is a wavelength in a free space at a given frequency and $\lambda_c$ stands for the cutoff wavelength for a given waveguide dimensions and waveguide mode.

11
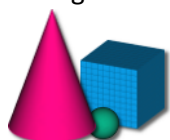
The frequency dependence can be investigated with *wr340_3GHz.QWpro* and *wr340_6GHz.QWpro* scenarios, where the WR-340 waveguide is excited with the $TE_{10}$ mode with two different frequencies. Load *wr340_3GHz.QWpro* project in the *QW-Modeller* and run the electromagnetic simulation in *QuickWave*. Open field visualisation window using button in *2D/3D Fields* tab of *QW-Simulator*. The display as in Fig. 18 should be seen.
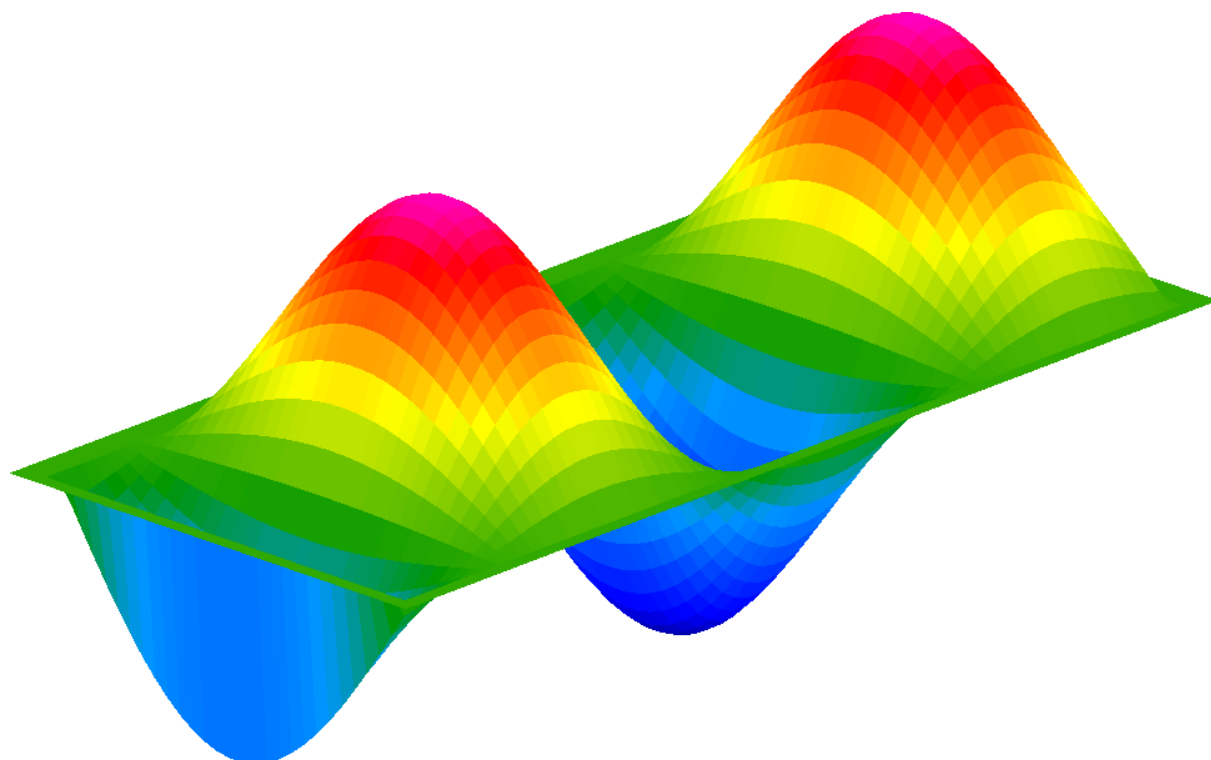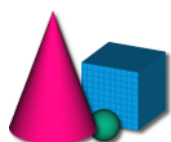


*Fig. 18 Ez component of electric field for TE10 mode in WR-340 waveguide at 3 GHz*

Load *wr340_6GHz.QWpro* in *QW-Modeller* and run the simulation for this project. In *QW-Simulator* open field visualisation window using button to obtain the display as in Fig. 19. The dependence of a wavelength in a waveguide on the wave frequency can be clearly seen.
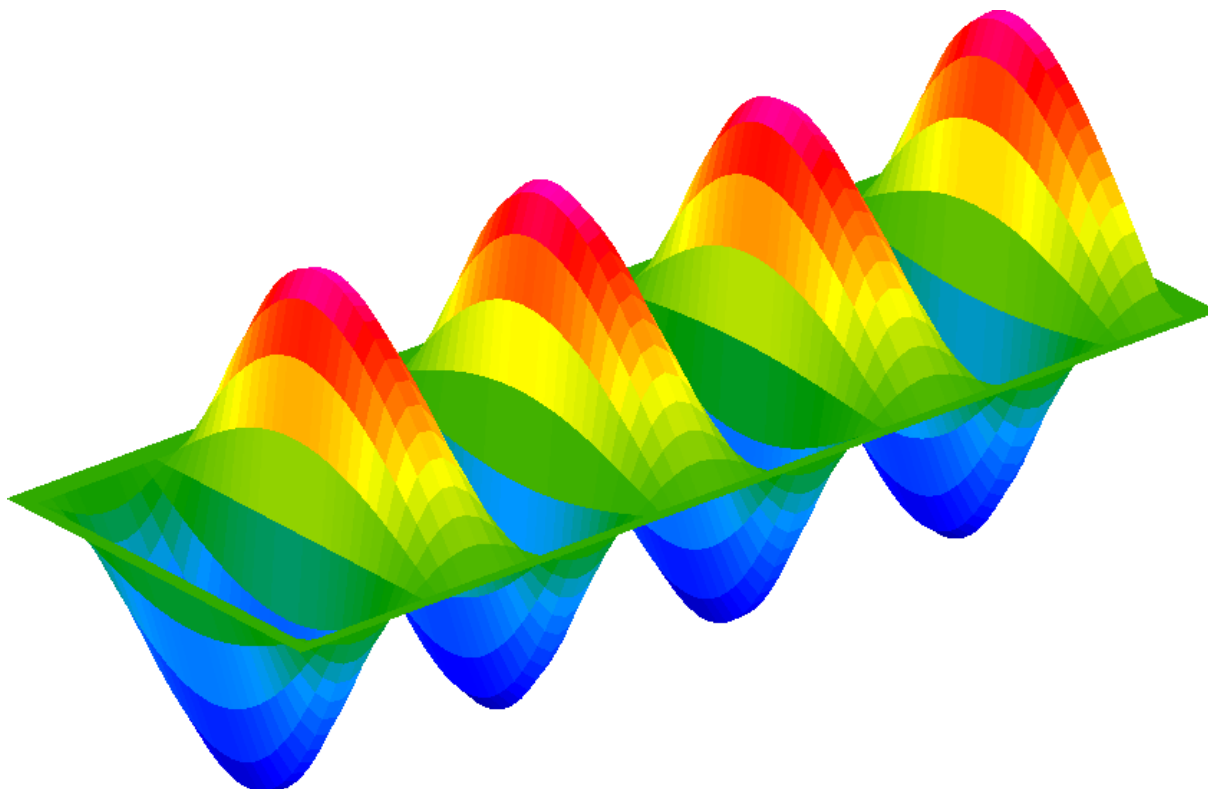
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*Fig. 19 Ez component of electric field for TE10 mode in WR-340 waveguide at 6GHz*

## Propagation below the cutoff frequency

It was stated above that the electromagnetic wave propagates in a waveguide at a chosen waveguide mode only if its frequency is above the cutoff. In this part of the tutorial, the behaviour of an electromagnetic wave at a frequency below cutoff value will be investigated.

The below relations describes a propagation constant in case of lossless medium filling the waveguide:

$$\gamma_z^2 = \beta^2 - \beta_\perp^2$$

If we consider a wave at a frequency below a cutoff value, the propagation constant is a complex number defined as follows:

$$\gamma_z = \alpha_z + j\beta_z = \sqrt{\left| \beta^2 - \beta_\perp^2 \right|}$$

$$\alpha_z = \sqrt{\left| \beta^2 - \beta_\perp^2 \right|}, \ \beta_z = 0$$

Thus the propagation constant becomes strictly imaginary, what means that the wave inside the waveguide will be attenuated. This can be confirmed with *wr340_1GHz.QWpro* project. In this scenario the excitation frequency is set to 1 GHz. As it was calculated earlier in this tutorial, the cutoff frequency for the dominant TE$_{10}$ mode in WR-340 waveguide equals to 1.73 GHz, which means that in the considered case the waveguide is excited at the frequency below cutoff value. To
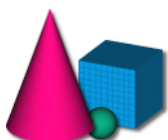
investigate this phenomenon, run the electromagnetic simulation in *QuickWave*. Open field

visualisation window using Fields . In this case, for better visualisation the field envelope calculation has been enabled (by pressing *'E'* button on the keyboard – for dynamic field observation 'E' button should be pressed). The display as in Fig. 20 should be seen.
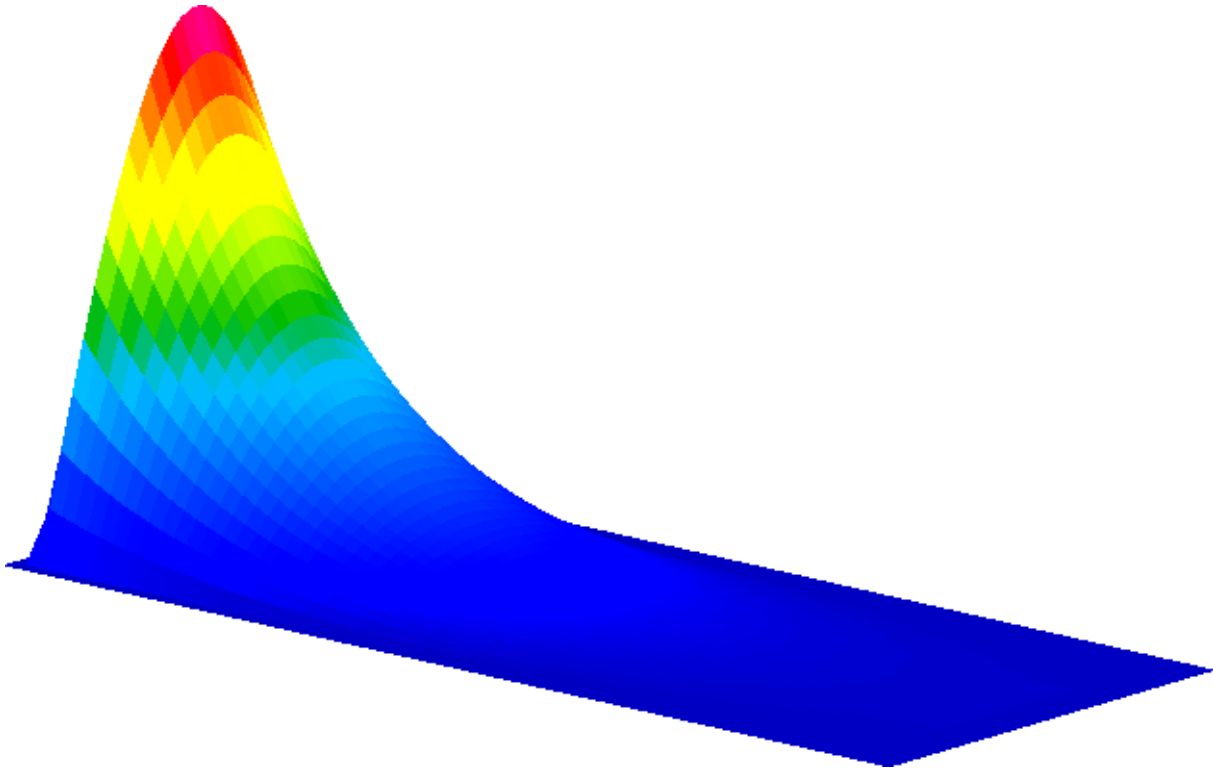


*Fig. 20 Ez component of electric field for TE10 mode in WR-340 waveguide at 1GHz*

It is clearly seen that the envelope of *Ez* component rapidly decreases along the waveguide. The field amplitude is proportional to $e^{-\alpha_z z}$ value.